SAMPLE QUESTION PAPER

MATHEMATICS

CLASS –XII (2013-14)

BLUE PRINT

Unit		VSA (1)	SA (4)	LA (6)	Total	
I.	Relations and Functions	1 (1)	4 (1)	–	5 (2)	10 (4)
	Inverse Trigonometric Functions	1 (1)	*4 (1)	–	5 (2)	
II.	Matrices	2 (2)	–	VB 6 (1)	8 (3)	13 (5)
	Determinants	1 (1)	4 (1)	–	5 (2)	
III.	Continuity and Differentiability	–	8 (2)	–	18 (4)	44 (11)
	Application of Derivatives	–	*4 (1)	6 (1)		
	Integrals	2 (2)	*4 (1)	6 (1)	18 (5)	
	Application of Integrals	–	–	6 (1)	–	
	Differential equations	–	8 (2)	–	8 (2)	
IV.	Vectors	2 (2)	*4 (1)	–	6 (3)	17 (6)
	3–dim geometry	1 (1)	4 (1)	*6 (1)	11 (3)	
V.	Linear Programming	–	–	VB 6 (1)	–	6 (1)
VI.	Probability	–	VB 4 (1)	*6 (1)	–	10 (2)

PEARSON edurite

SECTION–A

Question number 1 to 10 carry 1 mark each.

1. Write the smallest equivalence relation R on Set A = {1, 2, 3}.

2. If $|\vec{a}| = a$, then find the value of $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$

3. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined to x–axis at angles 30^0 and 120^0 respectively, then write the value of $|\vec{a} + \vec{b}|$

4. Find the sine of the angle between the line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and the plane $2x - 2y + z - 5 = 0$.

5. Evaluate $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.

6. If A = $\begin{pmatrix} 4 & 6 \\ 7 & 5 \end{pmatrix}$, then what is A. (adj. A)?

7. For what value of k, the matrix $\begin{pmatrix} 2k+3 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & -2k-3 \end{pmatrix}$ is skew symmetric?

8. If $\begin{vmatrix} \sin\alpha & \cos\beta \\ \cos\alpha & \sin\beta \end{vmatrix} = \frac{1}{2}$, where α, β are acute angles, then write the value of α+β.

9. If $\int_0^1 (3x^2 + 2x + k)dx = 0$, write the value of k.

10. Evaluate: $\int \cot x\,(\operatorname{cosec} x - 1)e^x dx$.

SECTION–B

Questions numbers 11 to 22 carry 4 marks each.

11. Let S be the set of all rational numbers except 1 and * be defined on S by a * b = a + b – ab, ∀ a, b εS. Prove that:

    a) * is a binary on S.

    b) * is commutative as well as associative. Also find the identity element of *.

12. If a + b + c ≠ o and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ = o, then using properties of determinants, prove that a = b = c.

13. Evaluate: $\int (2\sin 2x - \cos x)\sqrt{6 - \cos^2 x - 4\sin x}\ dx$

OR

Evaluate: $\int \frac{5x}{(x+1)(x^2+9)} dx$

14. Find a unit vector perpendicular to the plane of triangle ABC where the vertices are A (3, –1, 2) B (1,–1, –3) and C (4, –3, 1).

OR

Find the value of λ, if the points with position vectors $3\hat{i} - 2\hat{j} - \hat{k}$, $2\hat{i} + 3\hat{j} - 4\hat{k}$, $-\hat{i} + \hat{j} + 2\hat{k}$ and $4\hat{i} + 5\hat{j} + \lambda\hat{k}$ are coplanar.

15. Show that the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect. Also find their point of intersection.

16. Prove that $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \cot^{-1}3$.

OR

Find the greatest and least values of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$.

17. Show that the differential equation $x dy - y dx = \sqrt{x^2 + y^2}\, dx$ is homogeneous, and solve it.

18. Find the particular solution of the differential equation cos x dy = sin x (cos x – 2y) dx, given that y = 0 when $x = \frac{\pi}{3}$.

19. Out of a group of 8 highly qualified doctors in a hospital, 6 are very kind and cooperative with their patients and so are very popular, while the other two

remain reserved. For a health camp, three doctors are selected at random. Find the probability distribution of the number of very popular doctors.

What values are expected from the doctors?

20. Show that the function $g(x) = |x-2|$, $x \in R$, is continuous but not differentiable at $x = 2$.

21. Differentiate $\log\left(x^{\sin x} + \cot^2 x\right)$ with respect to x.

22. Show that the curves $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.

OR

Separate the interval $\left[0, \frac{\pi}{2}\right]$ into sub–intervals in which $f(x) = \sin^4 x + \cos^4 x$ is increasing or decreasing.

SECTION–D

Question numbers 23 to 29 carry 6 marks each.

23. Find the equation of the plane through the points A (1, 1, 0), B (1, 2, 1) and C (–2, 2, –1) and hence find the distance between the plane and the line $\frac{x-6}{3} = \frac{y-3}{-1} = \frac{z+2}{1}$.

OR

A plane meets the x, y and z axes at A, B and C respectively, such that the centroid of the triangle ABC is (1, –2, 3). Find the Vector and Cartesian equation of the plane.

24. A company manufactures two types of sweaters, type A and type B. It costs Rs. 360 to make one unit of type A and Rs. 120 to make a unit of type B. The company can make atmost 300 sweaters and can spend atmost Rs. 72000 a day. The number of sweaters of type A cannot exceed the number of type B by more than 100. The company makes a profit of Rs. 200 on each unit of type A but considering the

difficulties of a common man the company charges a nominal profit of Rs. 20 on a unit of type B. Using LPP, solve the problem for maximum profit.

25. Evaluate: $\int_0^1 x\left(\tan^{1-}x\right)^2 dx$

26. Using integration find the area of the region $\left\{(x,y): x^2+y^2 \le 1 \le x+\frac{y}{2}, x, y \in R\right\}$.

27. A shopkeeper sells three types of flower seeds A_1, A_2 and A_3. They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of three types of seeds are 45%, 60% and 35%. Calculate the probability.

a) of a randomly chosen seed to germinate.

b) that it is of the type A_2, given that a randomly chosen seed does not germinate.

OR

Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then two balls are drawn at random (without replacement) from Bag II. The balls so drawn are found to be both red in colour. Find the probability that the transferred ball is red.

28. Two schools A and B want to award their selected teachers on the values of honesty, hard work and regularity. The school A wants to award Rs. x each, Rs. y each and Rs. z each for the three respective values to 3, 2 and 1 teachers with a total award money of Rs. 1.28 lakhs. School B wants to spend Rs. 1.54 lakhs to award its 4, 1 and 3 teachers on the respective values (by giving the same award money for the three values as before). If the total amount of award for one prize on each value is Rs. 57000, using matrices, find the award money for each value.

29. A given rectangular area is to be fenced off in a field whose length lies along a straight river. If no fencing is needed along the river, show that the least length of fencing will be required when length of the field is twice its breadth.

MARKING SCHEME

SECTION–A

1. R = {(1,1), (2, 2), (3, 3)} 1

2. $2a^2$ 1

3. $\sqrt{2}$ 1

4. $\frac{1}{5\sqrt{2}}$ 1

5. $-\frac{\pi}{3}$ 1

6. $\begin{pmatrix} -22 & 0 \\ 0 & -22 \end{pmatrix}$ 1

7. $k = -\frac{3}{2}$ 1

8. $\frac{2\pi}{3}$ 1

9. k = –2 1

10. –cot x e^x + C 1

SECTION–B

11. a) let $a_1, a_2 \in S$

$\therefore a_1 * a_2 = a_1 + a_2 - a_1a_2$

Since $a_1 \neq 1, a_2 \neq 1 \Rightarrow (a_1 - 1)(a_2 - 1) \neq 0$

$\Rightarrow a_1a_2 - a_1 - a_2 + 1 \neq 0$ or $a_1 + a_2 - a_1a_2 \neq 1$

$\Rightarrow a_1 * a_2 \in S$ $\therefore *$ is a binary. 1

b) $a_2 * a_1 = a_2 + a_1 + a_1a_2 = a_1 * a_2 \Rightarrow *$ is commutative 1

also, $(a_1 * a_2) * a_3 = (a_1 + a_2 - a_1a_2) * a_3$

$= a_1 + a_2 - a_1a_2 + a_3 - a_1a_3 - a_2a_3 + a_1a_2a_3$

$a_1 * (a_2 * a_3) = a_1 * (a_2 + a_3 - a_2a_3)$

$= a_1 + a_2 + a_3 - a_2a_3 - a_1a_2 - a_1a_3 + a_1a_2a_3$

$(a_1 * a_2) * a_3 = a_1 * (a_2 * a_3)$ i.e. $*$ is associative 1

Let e be the identity,

Then a * e = a $\Rightarrow a$ + e – ae = a $\Rightarrow$ e (1–a) = 0 1

Since 1 – $a \neq 0 \Rightarrow$ e = 0.

12. $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$C_1 \to C_1 + C_2 + C_3 \quad \therefore \Delta = (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0$ 1

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1 \quad \therefore \Delta = (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix}$

$= (a + b + c)\,(-b^2 - c^2 + 2bc - a^2 + ac + ab - bc) = 0$ 1

$\Delta = -(a + b + c)\,(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$= -\frac{1}{2}(a + b + c)\,[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$ 1

Since $a + b + c \neq 0 \Rightarrow a-b = 0,\ b-c = 0,\ c-a = 0$

or a = b = c. 1

13. $I = \int (2\sin 2x - \cos x)\left(\sqrt{6 - \cos^2 x - 4\sin x}\right) dx$

$= \int (4\sin x - 1)\left(\sqrt{\sin^2 x - 4\sin x + 5}\right) \cos x\, dx$ 1

$= \int (4t - 1)\sqrt{t^2 - 4t + 5}\, dt$ where $\sin x = t$ ½

$= 2\int(2t-4)\sqrt{t^2-4t+5}\,dt + 7\int\sqrt{(t-2)^2+1}\,dt$ ½

$= 2\frac{(t^2-4t+5)^{3/2}}{\frac{3}{2}} + 7\left[\frac{t-2}{2}\sqrt{t^2-4t+5} + \log\left|(t-2)+\sqrt{t^2-4t+5}\right| + c\right]$ 1

$= \frac{4}{3}\left[\sin^2 x - 4\sin x + 5\right]^{3/2} + 7\frac{(\sin x-2)}{2}$

$\left[\sqrt{\sin^2 x - 4\sin x + 5} + \log\left|(\sin x - 2)\right| + \sqrt{\sin^2 x - 4\sin x + 5}\right] + C$ 1

OR

$I = \int\frac{5x}{(x+1)(x^2+9)}dx$

$\frac{5x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+c}{x^2+9} \Rightarrow A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{9}{2}$ 2

$\Rightarrow I = -\frac{1}{2}\int\frac{1}{x+1}dx + \frac{1}{2}\int\frac{x+9}{x^2+9}dx$ 1

$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\log(x^2+9) + \frac{3}{2}\tan^{-1}\frac{x}{3} + C$ 1

14. A vector perpendicular to the plane of ΔABC

$= \overrightarrow{AB} \times \overrightarrow{BC}$ ½

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & -5 \\ 3 & -2 & 4 \end{vmatrix} = -10\hat{i} - 7\hat{j} + 4\hat{k}$ 1+1

or $10\hat{i} + 7\hat{j} - 4\hat{k}$

$|\overrightarrow{AB} \times \overrightarrow{BC}| = \sqrt{100+49+16} = \sqrt{165}$ 1

$\therefore$ Unit vector $\perp$ to plane of ABC $= \frac{1}{\sqrt{165}}(10\hat{i} + 7\hat{j} - 4\hat{k})$ ½

OR

Let the points be A (3, –2, –1), B (2, 3, –4), C (–1, 1, 2) and D (4, 5, λ)

$\therefore \overrightarrow{AB} = -\hat{i} + 5\hat{j} - 3\hat{k},$

$\overrightarrow{AC} = 4\hat{i} + 4\hat{j} + 3\hat{k}$

$\overrightarrow{AD} = \hat{i} + 7\hat{j} + (\lambda + 1)\hat{k}$ 1½

A, B, C, D are coplanar if $\left[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}\right] = 0$ ½

$$\left[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}\right] = \begin{vmatrix} -1 & 5 & -3 \\ -4 & 3 & 3 \\ 1 & 7 & \lambda + 1 \end{vmatrix} = 0$$ ½

$\therefore$ 1(15+9) – 7 (–3 – 12) + (λ+1) (–3+20) = 0 1

$\Rightarrow \lambda = -\frac{146}{17}$ ½

15. Let, any point on the line $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ be P $(1 + 3\lambda, 1 - \lambda, -1)$ ½

and any point on line $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ be Q $(4 + 2\mu, 0, -1 + 3\mu)$ ½

If the lines intersect, P and Q must coincide for some λ and μ. ½

$\therefore$ 1+3λ = 4+2μ ..... (i)

1–λ = 0 ..... (ii)

–1 = –1 + 3μ ..... (iii)

Solving (ii) and (iii) we get λ = 1 and μ = 0 1

Putting in (i) we get 4 = 4, hence lines intersect. ½

$\therefore$ P or Q (4, 0, –1) is the point of intersection. 1

16. LHS = $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18}$ ½

$$= \tan^{-1}\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\cdot\frac{1}{8}} + \tan^{-1}\frac{1}{18}$$ 1

$$= \tan^{-1}\frac{3}{11} + \tan^{-1}\frac{1}{18}$$ ½

$$= \tan^{-1}\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{3}{11}\cdot\frac{1}{18}} = \tan^{-1}\frac{65}{195} = \tan^{-1}\frac{1}{3}$$ 1½

$= \cot^{-1} 3$=RHS ½

OR

$$(\sin^{-1}x)^2 + (\cos^{-1}x)^2 = \left(\sin^{-1}x + \cos^{-1}x\right)^2 - 2\sin^{-1}x\cos^{-1}x$$ ½

$$= \left(\frac{\pi}{2}\right)^2 - 2\sin^{-1}x\left(\frac{\pi}{2} - \sin^{-1}x\right)$$ ½

$$= \frac{\pi^2}{4} - \pi\sin^{-1}x + 2\left(\sin^{-1}x\right)^2$$

$$= 2\left[\left(\sin^{-1}x\right)^2 - \frac{\pi}{2}\sin^{-1}x + \frac{\pi^2}{8}\right]$$

$$= 2\left[\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$$ 1

$\therefore$ least value $= 2\left[\frac{\pi^2}{16}\right] = \frac{\pi^2}{8}$ 1

and greatest value $= 2\left[\left(\frac{-\pi}{2} - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right] = \frac{5\pi^2}{4}$ 1

17. $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \frac{\sqrt{x^2+y^2}}{x} = \frac{y}{x} + \sqrt{1+\left(\frac{y}{x}\right)^2} = f\left(\frac{y}{x}\right)$$ ½

$$x \to \lambda x,\ y \to \lambda y \Rightarrow \frac{dy}{dx} = \frac{\lambda y}{\lambda x} + \sqrt{1+\left(\frac{\lambda x}{\lambda y}\right)^2} = \lambda^0\left[\frac{y}{x} + \sqrt{1+\left(\frac{y}{x}\right)^2}\right]$$ 1

$$= \lambda^0 . f(y/x)$$

$\therefore$ differential equation is homogeneous.

let $\frac{y}{x}$ = v or y = vx $\therefore \frac{dy}{dx}$ = v + $x\frac{dv}{dx}$ ½

v+$x\frac{dv}{dx}$ = v+$\sqrt{1+v^2}$ or $\int \frac{dx}{\sqrt{1+v^2}} = \int \frac{dx}{x}$ 1

$\Rightarrow$ log$|$v + $\sqrt{1+v^2}$ $|$ = log cx $\Rightarrow$ v + $\sqrt{1+v^2}$ = cx 1

$\Rightarrow$ y + $\sqrt{x^2+y^2}$ = cx^2 1

18. Given differential equation can be written as

cot $x\frac{dy}{dx}$ + 2y = cos x or $\frac{dy}{dx}$ + 2 tan xy = sin x ½

$\Rightarrow$ Integrating factor = $e^{\int 2tanxdx} = e^{2logsec\,x}$ = sec² x. 1

$\therefore$ the solution is y. sec²x = $\int$ sin $x.sec^2x$ dx ½

= $\int secx.tanx\ dx$

y. sec²x = sec x + c 1

$\Rightarrow y$ = cos$x$ + c cos²x.

When $x = \frac{\pi}{3}$, y = 0 $\Rightarrow$ 0 = $\frac{1}{2} + \frac{1}{4}$C $\Rightarrow$ C = -2 ½

Hence the solution is y = cos$x$–2cos² x. ½

19. let x be the random variable representing the number of very popular doctors.

$\therefore$	**x :**	**1**	**2**	**3**	
P(x)		$\frac{{}^6C_1\,.\,{}^2C_2}{{}^8C_3}$	$\frac{{}^6C_2.\,{}^2C_1}{{}^8C_3}$	$\frac{{}^6C_3}{{}^8C_3}$	½
		$= \frac{3}{28}$	$= \frac{15}{28}$	$= \frac{10}{28}$	1½

It is expected that a doctor must be

- **Qualified**
- **Very kind and cooperative with the patients** **2**

20. $g(x) = |x-2| = \begin{cases} x-2, & x \geq 2 \\ 2-x, & x < 2 \end{cases}$ ½

LHL = $\lim_{x \to 2^-} (2-x) = 0$

RHL = $\lim_{x \to 2^+} (x-2) = 0$ and g (2) = 0 1

∴ g(x) is continuous at x = 2……………………………. ½

LHD = $\lim_{h \to 0} \frac{g(2)-g(2-h)}{h} = \lim_{h \to 0} \frac{0-(2-2+h)}{h} = -1$ 1

RHD = $\lim_{h \to 0} \frac{g(2+h)-g(2)}{h} = \lim_{h \to 0} \frac{(2+h-2)-0}{h} = 1$ ½

LHD ≠ RHD ∴ g(x) is not differentiable at x = 2 ½

21. Let y = log ($x^{\sin x} + \cot^2 x$)

$\Rightarrow \frac{dy}{dx} = \frac{1}{x^{\sin x} + Cot^2 x} \frac{d}{dx}(x^{\sin x} + \cot^2 x)$…………………….. 1

Let u = $x^{\sin x}$ and v = $\cot^2 x$.

∴ log u = sin$x$. log x , $\frac{dv}{dx} = 2 \cot x\,(-\mathrm{cosec}^2 x)$ ½

$\frac{1}{u}\frac{du}{dx} = \frac{\sin x}{x} + \log x.\cos x$

or $\frac{du}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \cos x \log x\right]$ ……………………………….. 1

∴ $\frac{dy}{dx} = \frac{1}{x^{\sin x} + \cot^2 x}\left[x^{\sin x}\left(\frac{\sin x}{x} + \cos x \log x\right) - 2 \cot x \,\mathrm{cosec}^2 x\right]$ 1 ½

22. Solving $xy = a^2$ and $x^2+y^2 = 2a^2$ to get $x = \pm a$

$\therefore$ for x = a, y = a and x = -a, y = -a

i.e the two curves intersect at P (a, a) and Q (-a, -a) 1

$xy = a^2 \Rightarrow x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = \frac{-y}{x} = -1$ at P and Q 1

$x^2+y^2 = 2a^2 \Rightarrow 2x+2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y} = -1$ at P and Q 1

$\therefore$ Two curves touch each other at P

as well as at Q. 1

OR

$f(x) = \sin^4 x + \cos^4 x$

$f'(x) = 4\sin^3 x\cos x - 4\cos^3 x\sin x$

$= -4\sin x\cos x(\cos^2 x - \sin^2 x)$

$= -2\sin 2x\cos 2x = -\sin 4x$ 1

$f'(x) = 0 \Rightarrow \sin 4x = 0 \Rightarrow 4x = 0, \pi, 2\pi, 3\pi, \ldots\ldots\ldots\ldots\ldots\ldots$

$x = 0, \frac{\pi}{4}, \frac{\pi}{2}$ 1

Sub Intervals are $\left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\therefore f'(x) < 0$ in $(0, \pi/4)$ $\therefore$ f(x) is decreasing in $(0, \pi/4)$ 1

And $f'(x) > 0$ in $(\pi/4, \pi/2)$ $\therefore$ f(x) in increasing in $(\pi/4, \pi/2)$ 1

SECTION – D

23. A vector ⊥ to the plane is parallel to $\overrightarrow{AB} \times \overrightarrow{BC}$ 1

$\therefore \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ -3 & 0 & -2 \end{vmatrix} = -2\hat{i} - 3\hat{j} + 3\hat{k}$ or $2\hat{i} + 3\hat{j} - 3\hat{k}$ 1½

∴ Equation of plane is $\vec{r}.(2\hat{\imath} + 3\hat{\jmath} - 3\hat{k}) = 5$ 1

$(\vec{r}.\vec{n} = \vec{a}.\vec{n})$

Since, $(2\hat{\imath} + 3\hat{\jmath} - 3\hat{k}).(3\hat{\imath} - \hat{\jmath} + \hat{k}) = 0$, so the given line is parallel to the plane. ½

∴ Distance between the point (on the line) (6, 3, -2) and the plane $\vec{r}$.

$(2\hat{\imath} + 3\hat{\jmath} - 3\hat{k}) - 5 = 0$ is

$$d = \frac{|12+9+6-5|}{\sqrt{4+9+9}} = \frac{22}{\sqrt{22}} = \sqrt{22}$$ 1½

Let the coordinates of points A, B and C be (a, 0, 0), (0, b, 0) and (0, 0, c) respectively. ½

∴ Equation of plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and 1

$\frac{a+0+0}{3} = 1$, $\frac{0+b+0}{3} = -2$ and $\frac{0+0+c}{3} = 3$ 1½

$\Rightarrow$ a = 3, b = -6 and c = 9

∴ Equation of plane is $\frac{x}{3} + \frac{y}{-6} + \frac{z}{9} = 1$ 1

or $6x - 3y + 2z - 18 = 0$ 1

which in vector form is

$$\vec{r}.(6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}) = 18$$ 1

24. let the company manufactures sweaters of type A = x, and that of type B = y. daily

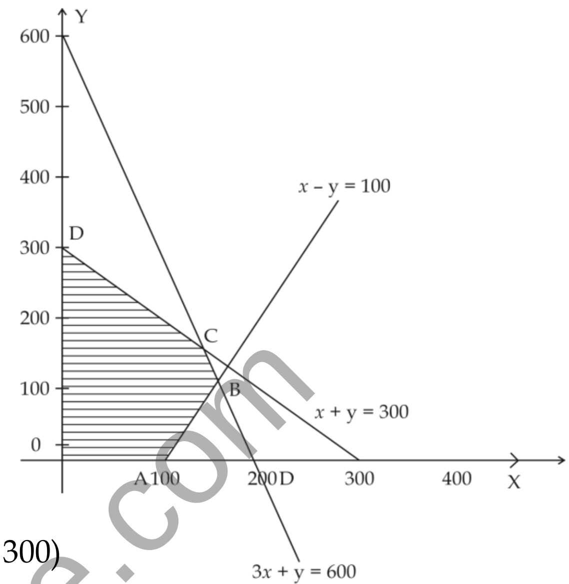
∴ LPP is Maximise P = 200x + 20y 1

s.t. x+y ≤ 300

360 x + 120y ≤ 72000

x – y ≤ 100 1½

x ≥ 0 y ≥ 0



Correct Graph 1½

Getting vertices of the feasible region as

A (100, 0), B (175, 75), C (150, 150) and D (0, 300)

Maximum profit at B

So Maximum Profit = 200 (175) + 20 (75)

= 35000 + 1500 1½

= Rs. 36500

25. let I $= \int_0^1 (tan^{-1}x)^2 . x\, dx$

$= \left[(tan^{-1}x)^2.\frac{x^2}{2}\right]_0^1 - \int_0^1 2tan^{-1}x.\frac{1}{1+x^2}\,\frac{x^2}{2}\,dx$ 1

$= \frac{\pi^2}{32} - \int_0^1 tan^{-1}x.\frac{x^2}{1+x^2}\,dx$ ½

$x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$ ½

$= \frac{\pi^2}{32} - \int_0^{\pi/4} \theta.tan^2\,\theta\, d\theta$

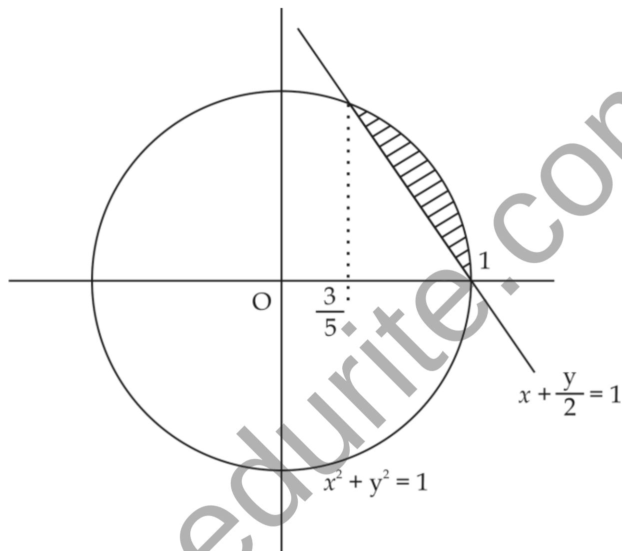
$= \frac{\pi^2}{32} - \int_0^{\pi/4} \theta.sec^2\,\theta\, d\theta + \int_0^{\pi/4} \theta.d\theta$ 1

$= \quad \frac{\pi^2}{32} - [\theta \tan\theta]_0^{\pi/4} \int_0^{\pi/4} \tan\theta \, d\theta + \left[\frac{\theta^2}{2}\right]_0^{\pi/4}$ 1

$= \quad \frac{\pi^2}{32} - \frac{\pi}{4} + [\log\sec\theta]_0^{\pi/4} + \frac{\pi^2}{32}$ 1

$= \quad \frac{2\pi^2}{32} - \frac{\pi}{4} + \frac{1}{2} \log 2 \text{ or } \frac{\pi^2 - 4\pi}{16} - \frac{1}{2}\log 2$ 1

26. Correct figure: 1



Correct Figure 1

Solving $x^2 + y^2 = 1$ and $x + \frac{y}{2} = 1$ to get $x = \frac{3}{5}$ and $x = 1$ as points of intersection

Required area $= \int_{3/5}^{1} \sqrt{1 - x^2} \, dx - \int_{3/5}^{1} (2 - 2x) dx$ 1

$= \quad \left[\frac{x}{2}\sqrt{1 - x^2} + \frac{1}{2}\sin^{-1}x\right]_{3/5}^{1} - [2x - x^2]_{3/5}^{1}$ 1

$= \quad \frac{\pi}{4} - \left(\frac{6}{25} + \frac{1}{2}\sin^{-1}\frac{3}{5}\right) - \left[1 - \frac{21}{25}\right]$ 1

$= \quad \left(\frac{\pi}{4} - \frac{2}{5} - \frac{1}{2}\sin^{-1}\frac{3}{5}\right)$ sq. u. 1

27. let E_1 : randomly selected seed is A_1 type $P(E_1) = \frac{4}{10}$

E_2 : randomly selected seed is A_2 type $P(E_2) = \frac{4}{10}$

E_3 : randomly selected seed is A_3 type $P(E_3) = \frac{2}{10}$ 1

(i) let A : selected seed germinates

$\therefore P(A/E1) = \frac{45}{100}, P(A/E_2) = \frac{60}{100}, P(A/E3) = \frac{35}{100}$ 1

$\therefore P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3).\,P(A/E_3)$ ½

$= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}$

$= \frac{49}{100}$ or 0.49 1

(ii) let A : selected seed does not germinate ½

$\therefore P(A/E_1) = \frac{55}{100}, P(A/E_2) = \frac{40}{100}, P(A/E_3) = \frac{65}{100}$ ½

$\therefore P(E_2/A) = \frac{P(E_2).P(A|E_2)}{P(E_1)P(A|E_1)+P(E_2)\,P(A|E_2)+ P(E_2)\,P(A|E_3)}$ ½

$= \frac{\frac{4}{10} \times \frac{40}{100}}{\frac{4}{10} \times \frac{55}{10} + \frac{4}{10} \times \frac{40}{100} + \frac{2}{10} \times \frac{65}{100}} = \frac{16}{51}$ 1

OR

Let E_1 : transferred ball is red.

E_2 : transferred ball is black. ½

A : Getting both red from 2nd bag (after transfer)

$P(E_1) = \frac{3}{7}$ $P(E_2) = \frac{4}{7}$ 1

$P(A/E1) = \frac{{}^5C_2}{{}^{10}C_2} = \frac{10}{45}$ or $\frac{2}{9}$ 1

$P(A/E2) = \frac{{}^4C_2}{{}^{10}C_2} = \frac{6}{45}$ or $\frac{2}{15}$ 1

$P(E_1/A) = \frac{P(E_1).P(A|E_1)}{P(E1).P(A|E_1)+P(E_2)\,P(A|E_2)}$ ½

$= \frac{\frac{3}{7}\cdot\frac{2}{9}}{\frac{3}{7}\cdot\frac{2}{9}+\frac{4}{7}\cdot\frac{2}{15}} = \frac{5}{9}$ 1+1

28. The three equations are $3x+2y+z = 1.28$

$$\left.\begin{aligned} 4x + y + 3z &= 1.54 \\ x + y + z &= 0.57 \end{aligned}\right\}$$ 1½

$\Rightarrow \begin{pmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1.28 \\ 1.54 \\ 0.57 \end{pmatrix}$ i.e $AX = B$ ½

$|A| = -5$ and $X = A^{-1}B$ 1

$A^{-1} = \frac{1}{5}\begin{pmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{pmatrix}$ 1

$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{pmatrix}\begin{pmatrix} 1.28 \\ 1.54 \\ 0.57 \end{pmatrix} = \begin{pmatrix} 0.25 \\ 0.21 \\ 0.11 \end{pmatrix}$ ½

$x = 25000$, $y = 21000$, $z = 11000$ 1½

29. let length be x m and breadth be y m.

∴ length of fence $L = x+2y$

Let given area = a $\Rightarrow xy = a$ or $y = \frac{a}{x}$

$\Rightarrow L = x + \frac{2a}{x}$ 1

$\frac{dL}{dx} = 1 - \frac{2a}{x^2}$ 1

$\frac{dL}{dx} = 0 \Rightarrow x^2 = 2a \quad \therefore x = \sqrt{2a}$ 1

$\frac{d^2L}{dx^2} = \frac{2a}{x^3} > 0$ ½

$\Rightarrow$ for minimum length $L = \sqrt{2a} + \frac{2a}{\sqrt{2a}} = 2\sqrt{2a}$ 1

$x = \sqrt{2a}$ and breadth $y = \frac{a}{\sqrt{2a}} = \frac{\sqrt{2a}}{2} = \frac{1}{2}x$ 1

$\Rightarrow x = 2y$ ½