Section - A

(Question No 1 Compulsory and Attempt five other questions)

Question 1

1. If \( \log_{10} x = a \), find the value of \( 10^{2a-1} \) in terms of \( x \).
2. If \( p(11,r) = p(12,r-1) \) find \( r \)
3. \( f: \mathbb{R} \rightarrow \mathbb{R} \) be defined as \( f(x) = x^2+1 \) find \( f^{-1}(-5) = x \)
4. Find the angle between the lines whose direction ratios are proportional to 4,-3,5 and 3,4,5.
5. show that \( f(x) = (x-1)e^{x}+1 \) is an increasing function on \((-\pi/2,\pi/2)\)
6. Evaluate \( \lim_{x \to 0} \frac{\cos x - \cos y}{\cot x - \cot y} \)
7. Show that the origin is equidistance from the line \( 4x+3y+10 = 0; 5x-12y+26=0 \) and \( 7x+24y = 50 \)
8. Find the sum and product of the roots \( 3x^2-4x+9 \)
9. Find two positive numbers whose difference is 12 and whose A.M exceeds the G.M by 3.
10. Find the antilogarithm of a) 1.23 b) 2.5647

Question 2:

1. Evaluate the following \( \log_{10}10+\log_{10}100+\log_{10}1000+\log_{10}10000 \) \[5m\]
2. If \( a^x = b^y = c^z \) and \( x,y,z \) are in G.P prove that \( \log_a a = \log_b b \) \[3m\]
3. \( a^2 = b^2 = c^2 = d^4 \), prove that \( (abcd) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \) \[3m\]

Question 3:

1. Find the value of \( x \) and \( y \) if \( \frac{x-1}{3+i} + \frac{y-1}{3+i} = i \) \[5m\]
2. Find the modulus and argument of the following complex number and write in polar form of \( \frac{1+3i}{1-2i} \) \[5m\]

Question 4:

1. If \( a,b,c \) are in A.P, prove that \( a^2+c^2+4ac = 2(ab+bc+ca) \) \[5m\]
2. Find the sum to \( n \) terms of the sequence \( \left(x+\frac{1}{x}\right)^2, \left(x^2+\frac{1}{x^2}\right)^2, \left(x^3+\frac{1}{x^3}\right)^2, \ldots \) \[3m\]
3. Find the number of permutation of different things taken r at a time such that two specified things occur together [2m]

Question 5:
1. Sketch the graph of the given function \( y = 2 \cot 2x \) [4m]
2. Prove that \( 2 \sin \frac{\pi}{6} + \csc \frac{7\pi}{6} \cos \frac{\pi}{3} = \frac{3}{2} \) [3m]
3. Find the radian measure corresponding to degree measures:
   a) \( 30^0 \)
b) \(-56^0\) [3m]

Question 6:
1) Find the value of the function \( f(x) = 1 + \alpha x \), \( \alpha \neq 0 \) is the inverse of the function. [4m]
2) For any set A, B, C, D prove that: \((A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)\) [3m]
3) Find the center and radius of the circle \( x^2 + y^2 - 4x + 6y = 12 \) [3m]

Question 7:
1. Evaluate \( \int \sec^3 x \tan x \, dx \) [5m]
2. Find the image of the point \((8, -12)\) with respect to the line mirror \( 4x + 7y + 13 = 0 \) [5m]

Question 8:
1. Find the domain of the function \( f(x) = \frac{1}{\log 10(1 - x)} + \sqrt{x^2 + 2} \) [5m]
2. If \( f(x) = \cos \left( \log x \right) \), then find the value of \( \frac{1}{2} \left( f \left( \frac{x}{y} \right) + f(xy) \right) \) [5m]

Section - B
(Assert any two questions)

Question 9:
1. Determine the values of \( x \) for which \( f(x) = x^x \), \( x > 0 \) is increasing or decreasing [4m]
2. Find the derivative of the function using first principles: \( \cot \sqrt{x} \) [3m]
3. Discuss the differentiation of \( f(x) = x \left| x \right| \) at \( x = 0 \) [3m]

Question 10:
1. The mean and standard deviation of 15 observations are found to be 8 and 10 respectively. On rechecking it was found that an observation 4 was incorrect. Calculate the correct mean and standard deviation in any of the cases. [5m]
2. Calculate the median of the given table [5m]

<table>
<thead>
<tr>
<th>variables</th>
<th>10-20</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>46</td>
<td>32</td>
<td>36</td>
<td>60</td>
<td>54</td>
<td>62</td>
</tr>
</tbody>
</table>

Question 11:
1. Find the average due date of the following bills: [5m]
<table>
<thead>
<tr>
<th>Amount of the bill (Rs)</th>
<th>Date of acceptance of the bill</th>
<th>Period of the bill</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>10-1-2013</td>
<td>3 months</td>
</tr>
<tr>
<td>4500</td>
<td>12-2-2013</td>
<td>2 months</td>
</tr>
<tr>
<td>3000</td>
<td>15-3-2013</td>
<td>1 months</td>
</tr>
<tr>
<td>2000</td>
<td>20-3-2013</td>
<td>2 months</td>
</tr>
<tr>
<td>1000</td>
<td>10-5-2013</td>
<td>3 months</td>
</tr>
<tr>
<td>750</td>
<td>13-6-2013</td>
<td>2 months</td>
</tr>
</tbody>
</table>

2. Find the median of the following distribution

\[ x: \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \\
 f: \quad 15 \quad 35 \quad 60 \quad 84 \quad 96 \quad 127 \quad 198 \]